

The Nature of the divergences in Perturbation Theory

Two questions

- (1) are renormalized quantities themselves finite or just a divergent sum of finite terms?

Model calculations ($\lambda\phi^3$ scalar theory) by
Hurst & Thirring (1952 & 1953) showed
 n^{th} order term $\sim \lambda^n n^{n/2} \cdot n^{-2}$
 \uparrow \uparrow
no of terms power law on each term

So series diverges.

But n^{th} order term could be conditionally convergent to zero by cancellations of signs.

Probably series is asymptotic.

Typically even ^{order} involved is smaller than last term calculated, but after a while this last term starts to rise, since series actually diverges as $n \rightarrow \infty$.

Two arguments for asymptotic nature of expansion:

- (1) Dyson: $e \rightarrow ie$ series cannot be expected to converge or indeed to correspond to "a well-defined function" due to fact that repulsion of opposite charges would lead to an "exploding disintegration of the vacuum by spontaneous polarization".

- (2) Excellent argument between theory & experiment which shows series asymptotic (Hurst (1952))

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(2) Are all renormalization constants finite?

They may actually be finite and appear infinite because the relevant functions are not analytic.

e.g. $e^{-e^2 L} = 1 - e^2 L + \frac{1}{2} e^4 L^2 - \dots$

as $L \rightarrow \infty$, L.H.S. $\rightarrow 0$ but each term in the formal perturbation series is infinite.

Hog's theorem shows that Dyson's V-operator, for finite times, used by him to link the Heisenberg or interaction representations are not exact.

Barton (1963) says "With due fact in mind the occurrence of formal divergences is to be expected, and should in no way surprise us".

Barton (1968) comments "We may now wonder why, in spite of its non-existence, the interaction picture leads, at least in perturbation theory, to reasonable results. The 'mystery' results, in a sense, the underpinning manipulations of ordinary perturbation quantum mechanics.

Hog's theorem says sets of operators referring to free fields & interacting fields cannot belong to equivalent representations of the canonical equal-time commutation relations. The existence of non-equivalent representations is linked to the fact dimensionality of the Hilbert space is non-denumerably infinite with spaces non-separable. (cf also Schwinger's theory developed in late 1960's.)

Transferring
(18)

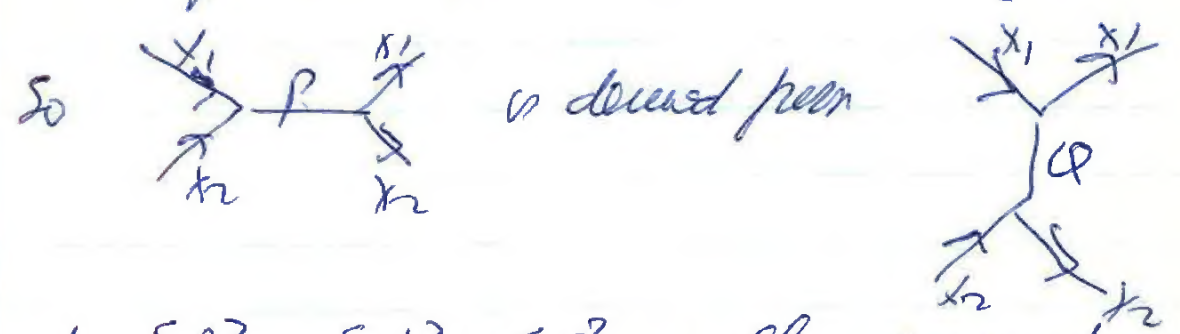
Partial bootstraps

(1) $P = \pi \pi(P)$

(2) $N = \pi N(N)$
 $\pi = N N(\pi)$

(3) $N = \pi N(N^*)$ Reciprocal bootstrap
 $N^* = \pi N(N)$

In general $\{P, P_2, \dots\} = X_1, X_2, \dots \{Q, Q_2, \dots\}$



and $\{P\}, \{X\}, \{Q\}$ are all same set.

~~A = B + C~~
Anomalies

$$\left. \begin{aligned} A &= B + C \\ B &= A + C \end{aligned} \right\} \begin{aligned} &A \text{ is part of } B \\ &\text{and } B \text{ is part of } A. \end{aligned}$$

or $A = A + B$ — A is part of itself.

Resolution of anomalies

Energy in incident system is used to "create" outgoing particle, not to "release" what is already there.

e.g. $X + A = (B + C) + X$

or not to interpret as A being broken up into B & C by collision with particle X.

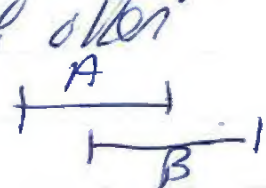
Compare with other philosophers

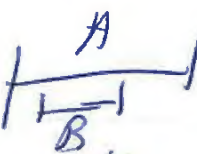
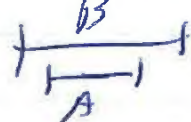
- (1) Leibniz the stones analogy with principle of sufficient reason
 "Nature is as it in fact is because this is the only possible world consistent with itself."
 i.e. nothing in nature is arbitrary.
 But for Leibniz there are many possible worlds allowed by mere consistency. Existence is not itself determined by consistency but is caused by God choosing best possible world.
 (But cf. Russell on Leibniz's metaphysics)

- (2) Anaxagoras claims every substance contains every other substance.

But A's view is essentially a containment model.
 "seeds contain portions of all seeds".

cf. 2 sets A & B may each include subsets of each other



But if  then  is impossible.

Also for Anaxagoras atoms are impossible

But this is ruled down to view that substances can change their form.

cf. Thales, Anaximenes, Anaximander and especially Heraclitus.

Anaxagoras is ruled down to Empedocles or Leucippus & Democritus.

(2) Analogies with Eastern Philosophy
and in Hinduism & Buddhism
cf. "The Tao of Physics" by Capra (1975)

(3) Whitehead Stapp (1971) draws analogy
with a web philosophy of interlocking
processes - cf. Whitehead's Process and
Reality.

Shortcomings of the Bootstrap

1. It makes life very difficult for physicists
2. Partial bootstrap may be impossible
to satisfy - we may have to include
all particles at once
3. It is very untestable, and so unscientific,
because of computation gap
4. Bootstrap does not include creation
of photon - Chew claims these particles
are associated with the process of
measurement, so should be treated
differently.
But what about the other leptons
- the quarks & the neutrinos.
5. Chew notes some would metaphysicist
claims. The very complete bootstrap
would demand for self-consistency
"confronting the elusive concept of
measurement and possibly even of consciousness"
(1968, Science)

The Ultimate Nature of Matter

1. Bootstrap Models

$$x_1 = \begin{cases} x_1 \\ x_0 + x_2 \\ \vdots \end{cases}$$

$$x_2 = \begin{cases} x_2 \\ x_0 + x_2 \\ \vdots \end{cases}$$

2. Thales Fundamentalism

$$x_1 = Q_1 + Q_2 + \dots$$

$$x_2 = Q_1 + Q_2 + \dots$$

on
single
element
model
of quarks or leptons

3. Abelianization Fundamentalism

$$x_1 = x_1(F)$$

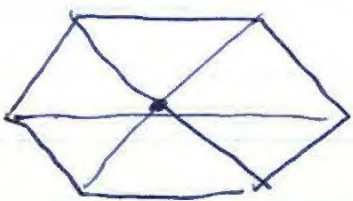
$$x_2 = x_2(F)$$

cf Heisenberg's
wicked red dog

cf Probs on a string
(Rattler 1971)

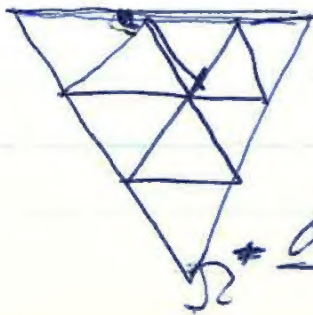
4. Rationalized Atomism

Analogy between Plato's theory in Timaeus
of building regular solids from 2 sets
of triangles and the SU(3) symmetry
patterns being built up out of
simple triangular representations.



octet

or



N^* sextet

Σ^* decuplet

Conclusions

1. Analysis of correspondence relations

Emphasizes the correspondence aspect in theories.

Our analysis has been in terms of
of a directional shift followed by
a "stretching" in the direction indicated
by a polarizing phenomenon or by

a polarizing property of some model of
the old theory.

(This elaborates on the point made alluded
in my Synthesis paper.)

2. Role of surplus structures

(a) ~~The way of a~~ Reformulation involves
changing the surplus structure
in which the theory is embedded.

(b) "Stretching" may involve introducing
a new axiom which operates in
the surplus structure (of analyticity
postulate)

(c) In some cases surplus structure comes
to be afforded ontological reference
(cf Fock space) but in other
cases this does not happen
(cf Analytic S-matrix theory).

(d) Emphasis on purely mathematical considerations
is a feature of modern theoretical

Physics - cf Einstein & Durr

(a) could have both been stressed by Zohar using
as examples. Maxwell's introduction of displacement
Einstein's use of non-commuting
Weyl's interpretation of his
transformation eqs. in terms of a
group construction
Durr's interpretation of -ve energy
states as positions etc.

Quotation from Durr (1931)

"Advances in physics is to be associated with
a continual modification & generalization
of the axioms at the base of the mathematics
rather than with a logical development
of any one mathematical scheme on a
fixed foundation" [as was expected
in the last century]
cf. Her. Evidences pointing to relativity
non-commutative algebra in QM

3. The Problem of the Computation Gap - The Floating Model

If an approximate calculation disagrees
with experiment we do not know
whether to desert the model
tollens at the required theory or
at the approximation
In the case of atomic or molecular
physics we have some confidence in the

in the underlying theory because there are simple unsolvable problems, such as the hydrogen or helium atom, or the hydrogen molecule, which can be solved very accurately so these predictions really do test the theory and not the theory + approximation.

We can now argue that if in a more complicated problem (in many nuclear physics & chemistry problems) a scheme of approximation gives results in agreement with experiment then we may believe the approximation has picked out the essential relevant features of some complicated dynamical situation, i.e. we can justify our approximation a posteriori in virtue of its success.

But in particle physics of strong interactions there are no simple solvable problems - this is the message of the bootstrap.

Hence our approximation models are not anchored in any secure underlying theory - in this sense they may be said to float.

(Contrast Post's sense of floating models in which approximate models do not give much deviation - they float at both ends (theoretical & experimental))
But such models are only objectionable if they are not allied with a requirement that the model be in accord with experiment is desirable (theoretically)

in a simple way - of effects $SU(3)$ in
nuclear physics
where interactions which break the symmetries
is specified to be a function of the generators
of the symmetry. so there is no mixing
of different irreducible representations;

4. The Failure of Scientific Method

The bootstrap philosophy, if correct, would
point to a fundamental impasse
for scientific method as we know it.

The success of scientific methods depends
on the possibility of being able to
isolate phenomena and of being able
to disregard the enormous complexity
of real-life situations.

The bootstrap philosophy would tell
us that in the realm of hadron
dynamics this approach is no
longer possible. As a parallel one
can think what celestial mechanics
would have been like if the planetary
system was not susceptible to perturbation
theory.

The bootstrap philosophy is essentially
one of despair and frustration
although Chew himself sees things just
the other way around (1970)

"I would find it a crushing disappointment if in 1980 all of hadron
physics could be explained in terms of a few arbitrary entities
- We should then be in essentially the same posture as in 1930
... to have learned so little in half a century would to me
be the ultimate frustration"

TABLE of the ELEMENTARY PARTICLES

BOSONS		FERMIONS	
<p>150 π (pion)</p> <p>500 K (kaon)</p> <p>550 η</p> <p>+ resonances</p> <p>750 ρ</p> <p>800 ω</p> <p>3,100 ψ</p> <p>3,700 ψ</p>	<p>\longleftrightarrow HADRONS \longleftrightarrow</p>	<p>nucleons { proton 950 neutron 950</p> <p>hyperons { Λ 1100 Σ 1200 Ξ 1300 Ξ 1300 Ξ 1650</p> <p>+ resonances Δ (236)</p>	
<p>o photon</p> <p>o graviton (?)</p> <p>2000 ^{10,000} W-particle (?)</p>		<p>LEPTONS { ν_e (neutrino) 0 ν_μ (neutrino) 0 e (electron) $\frac{1}{2}$ μ (muon) 100</p>	
<p>Quarks (?) 5000 25,000</p>			

(Rest energies in MeV rounded to nearest 50 MeV)

The Analytic S-Matrix

Two strands in research programme of the analytic S-Matrix.

- (1) What should fundamental study of elementary particles refer? — to S-Matrix
- (2) a new non-perturbative method of calculating the S-Matrix.

Heisenberg (1943) introduced S-Matrix (cf. Heisenberg (1937))
by asking 2 questions

- (1) If a "complete" theory involves a fundamental length what features of existing theories would survive (cf. Einstein's approach to special relativity)
- (2) Should not a theory anyway restrict itself to what can actually be observed (cf. HS approach to matrix mechanics)

Heisenberg answered both questions with his S-Matrix which provides 2 sorts of information

- (1) Scattering & reaction cross-sections
- (2) Bound states and resonances (short lived unstable particle states) related to singularities in the S-Matrix

Non-Perturbative Calculations of the S-Matrix

Success of Q.F.D. due to small value $e^2/\hbar c$
 (nevertheless vertex particles
 in "closed")
 No colour perturbation Theory.

For hadron physics (the vast majority of ~ 500 particles)
 we are dealing with strong interactions
 so perturbation theory does not apply.

Four Basic Interactions

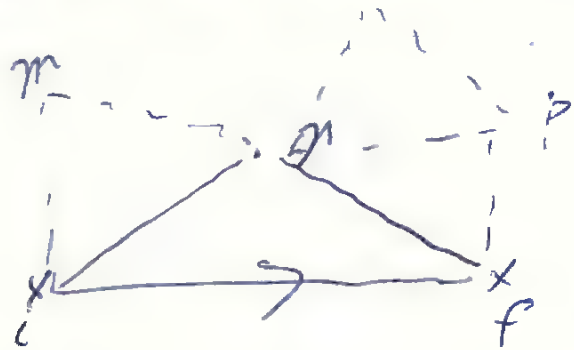
Strong \longleftrightarrow chemical bond
 Electromagnetic \longleftrightarrow chemical high explosive
 weak
 gravitational

(Table of the Particles)

Approximation schemes

1. Tamm Duffin: Limited no. particles on arbitrary no. of states
2. Tomonaga: Arbitrary no. of particles on a limited no. of states.
 Both deal with vertex particles
 Much more successful approach to low energy PN scattering achieved by
3. Chew-Low-Wick: Low energy scattering

amplitudes for a real (unnormalized) physical process in terms of scattering amplitudes for all real processes which could connect with both the initial & final states



Sehne (1955) showed that Chew-Low model was an example of a dispersion relation and was connected with analytic properties of the S -matrix.

Dispersion Relations

Dispersion relations approach to calculating the S -matrix involves the following sequence of ideas.

- (1) Consider S -matrix elements as functions of energy (& other variables such as momentum transfer) and now allow energy to assume complex values.
- (2) Assume S -matrix is an analytic function except for certain singularities (by Cauchy's theorem a bounded analytic function with no singularities is identically a constant.)

- (3) Use Cauchy's Theorem to relate scattering amplitudes to the singularity structure (position of singularities & behaviour in their neighbourhood - residues at poles, discontinuities across branch cuts).
- (4) Use unitarity & crossing principles to locate part of the singularity structure
- (5) We assume there are no other singularities than those demanded by unitarity & crossing — Principle of Maximal analyticity of the first kind or the Mandelstam conjecture.

(6) We now have fixed the relation.

Non-relativistic

(14)



$$\text{or } \left. \begin{array}{l} Y = X \\ X = Y^2 \end{array} \right\} Y = 0 \text{ or } 1$$

- (7) We look for possible ambiguities in solutions of integral equations and seek to remove them by a principle which demands certain singularities. This leads to the Chen-Frosticki Principle of Maximal analyticity of the 2nd kind.

then expressed in terms of analytic in
complex angular momentum plane. 10.
every particle pole is a Regge pole 10. lies
on a Regge trajectory in the complex angular
momentum plane.

(In our model if $\alpha(0) = 0$ no quadratic under
reflection in the origin $\Rightarrow \gamma = 0$)

this principle leads to the ~~Chew-Frautschi~~

Chew-Frautschi Bootstrap Hypothesis (1961)

there are no "arbitrary" particle poles which
all poles must be identified with the
elementary particles (aristocrats)

all poles are determined by self-consistency
and are dynamical in origin in view
of the fact that the location of the
poles depends on the strength of the interaction.
(nuclear democracy)

(Note: The principle of Natural Cuts of
all 2nd kind connects to ensuring
absence of C.P.D. poles in Chew-Low
-model. The source of the ambiguity
is that the Mandelstam expansion tells
us nothing about asymptotically at ∞ ,
i.e. asymptotic behaviour. C.D.
elimination of C.P.D. ambiguity is
equivalent to ensuring vanishing of
phase shifts at high energy)

How

Principle of Maximum Strength The still remained the question of whether all arbitrary parameters could be separated from the study.

In 1962 Chew & Frautschi introduced a Principle of Maximum Strength to fix the overall strength of the interactions.

(This may not be required as a separate assumption)

The final objective of the Hadron bootstrap is to have no arbitrary parameters except a dynamical constant & to fix the scale of the Hadron masses.

There are 3 possibilities:

- (1) several sets of particles that satisfy the bootstrap
- (2) no set of particles that satisfy the bootstrap
- (3) there is a unique set of particles that satisfy the bootstrap and these are the particles observed as Nature.

The last possibility we may call the Chew - Frautschi Hypothesis.

Transparency
(15)

Singularities from Unitarity & Crossing

Resonance amplitudes \rightarrow Im Scattering amplitude

\rightarrow singularity structure of particle poles & normal threshold branch points.

Crossing states that no scattering amplitude can be related to another by analytic continuation to complex values of the energy or momentum transfer (in the simplest case of 2-body elastic scattering) Unitarity now forces singularities in these closed channels which appear as singularities in the direct channel at complex values of the energy and momentum transfer arguments

Analyticity & Causality

Transparency
(16)

We can derive some analytical properties from causality in classical physics - e.g. the Kramers-Kronig dispersion relations in optics.

To prove this in QFT we can try to derive analyticity from micro causality or equivalently from the canonical commutation relations

$$[\phi_1(x_1, t_1), \phi_2(x_2, t_2)] = 0 \quad \text{if } (x_1, t_1) \rightarrow (x_2, t_2) \text{ is space-like interval}$$

But we could equally derive analyticity in different base domains to reflect full power of dispersion relations approach.

16

We want complete information about analytic properties in all variables

The Status of the Mandelstam Conjecture

Mandelstam (1958) introduced his conjecture by two ~~arguments~~ implicit arguments.

- (1) Let us be guided by successes of such dispersion relations as can be derived from field theory (e.g. forward scattering of π -N system)
- (2) Let us use what we can know from examination of analytic properties of Feynman graphs (Mandelstam had studied some 4th order Feynman graphs in detail for this purpose)

In his 1958 paper Mandelstam went beyond his conjecture to assume that the singularities dictated by unitarity would be such as to allow a specific representation (in ~~terms~~ terms of a double dispersion relation) — the Mandelstam representation

But, Mandelstam himself showed that his specific representation was not generally true in perturbation theory (1959) nor could it be proved from axiomatic field theory for cases where perturbation theory suggested it should hold (1960).

However, Mandelstam representation is true for non-relativistic potential scattering (by Yukawa with e.g.) [Blankenbelder, Goldberger, Khuri, & Treiman (1960) & Regge (1959)]

Nordstrom Remick regarded his conjecture as just that, a conjecture as to how field theories should behave. He did not subscribe to a sumative S-matrix theory because

- (1) S-matrix contains less than can be potentially measured
- (2) Analyticity assumptions as axioms "or rather artificial" (1962 review)

But Chew & others (Frankel, Staff, Landau) took up enthusiastically the idea that dispersion relations could replace field theory (originally proposed by Gell-Mann at Rochester Conference in 1956)

Nordstrom Conjecture would play the role of a postulate of axiom (cf. Hoang 1946)

Chew regards the formal analytic postulate as the mathematical counterpart of a fundamental principle of modern mathematics

In his 1966 book The Analytic S-matrix Chew emphasizes the mathematical aspect of the postulate

"In a deep sense physics is based on analytic functions. It is pointless to seek a logical origin for this circumstance. Physical theory cannot be based on logic; it is always a matter of guesswork based on observation of Nature. One cannot, for example, argue that it is

logical for classical mechanics to be expressible through second order differential equations. This is simply in the sense that works."

(cf. Forster who had asked physics to deal with non-analytic functions!)

So I have shown an axiom which is concerned essentially with the "surface structure"

(note Stapp for discussed links between macro causality and normal analytic structure in the physical region).

But macro-analyticity postulate is still required to extend the primitive domain of analyticity so covered by the macro causality principle).

The Chew-Nordstrom Heuristic Strategy

Temporary

(17)

- (1) Nordstrom derives a property of analyticity by expanding on approximation, models of field theory (12th order perturbation theory)
- (2) Nordstrom conjectures the property may be true of complete theory
- (3) Chew now reinstates the expectation as being "model-independent" by fiat and takes it as an axiom for

a new theory which may or may not
be equivalent to the old theory.

(17) Structure form - identical in tempore

T is $R \cup T$
 A is Feynman Perturbation affixed
 T_1 is a class of Feynman diagrams
 P is analytic property of these
 diagrams
 C is dispersion relations connecting
 observable quantities which are
 verified experimentally
 $T'(P)$ is a maximal analytic function
 theory

(Another example of this strategy is
Current algebra (Jell-Mann (1962))